

# PUNJAB PUBLIC SERVICE COMMISSION

**Competitive Examination (June-2017) for Recruitment of Lecturer Math in the Department of Technical Education & Industrial Training, Govt. of Punjab**

**READ INSTRUCTIONS BEFORE FILLING ANY DETAILS OR ATTEMPTING TO ANSWER THE QUESTIONS.**

**Candidate's Name** \_\_\_\_\_

**Father's Name** \_\_\_\_\_

**Date of Birth**             

DD                      MM                      YYYY

**Category Code\***   

(\*as given in the admit card)

**OMR Response Sheet No.** \_\_\_\_\_

**Roll No.** \_\_\_\_\_

Booklet No.

**Candidate's Signature** (Please sign in the box)

## INSTRUCTIONS

<p>1. The candidate shall NOT open this booklet till the time told to do so by the Invigilation Staff. However, in the meantime, the candidate can read these instructions carefully and subsequently fill the appropriate columns given above in CAPITAL letters. The candidate may also fill the relevant columns (other than the columns related to marking responses to the questions) of the Optical Mark Reader(OMR) response sheet, supplied separately</p>	<p>9. The candidates shall be responsible to ensure that the responses are marked in correct manner and any adverse impact due to wrong marking of responses would be the responsibility of the respective candidate. The following are some of the <b>examples of wrong marking</b> of responses on the OMR response sheet.</p> <div style="text-align: center;"> </div>
<p>2. <b>Use only blue or black ball point pen</b> to fill the relevant columns on this page. Use of fountain pen may leave smudges which may make the information given by the candidate here illegible.</p> <p>3. The candidate shall be liable for any adverse effect if the information given above is wrong or illegible.</p>	<p>10. The candidates, when allowed to open the question paper booklet, are advised to check the booklet to confirm that the booklet has complete number of pages, the pages printed correctly and there are no blank pages. In case there is any such error in the question paper booklet then the candidate should immediately bring this fact to the notice of the invigilation Staff and obtain a booklet of the same series as this one.</p>
<p>4. The candidate must fill all the columns given above on this page and sign at the appropriate place.</p> <p>5. <b>Each candidate is required to attempt 100 questions in 120 minutes</b>, except for orthopaedically/visually impaired candidates, who would be given 40 minutes extra, by marking correct responses on the OMR sheet which would be supplied separately to the candidates</p>	<p>11. The serial number of the new booklet should be entered in the relevant column of the OMR. The candidate should request the Invigilation Staff to authenticate the change in serial number of question booklet by obtaining the initials of the Staff on the corrected serial number of the question booklet</p>
<p>6. <b>The candidate must write the following on the OMRs sheet:</b>                  (a)Serial number of OMR sheet supplied to him/her for marking the responses to the questions.                  (b)Serial number of the question booklet                  Failure to do so may lead to cancellation of candidature or any other action which the Commission may deem fit.</p>	<p>12. The question paper booklet has 20 pages.</p> <p>13. <b>Each question shall carry three marks.</b></p>
<p>7. <b>The candidate should darken the appropriate response to the question by completely darkening the appropriate circle/oval</b> according to his/her choice of response i.e. a, b, c or d in the manner shown in the example below.</p> <div style="text-align: center;"> </div>	<p>14. <b>There are four options for each question and the candidate has to mark the most appropriate answer on the OMR response sheet using blue or black ball point pen.</b></p>
<p>8. Partly darkening the circle/oval on the OMR response sheet or using other symbols such as tick mark or cross would not result in evaluation of the response as the OMR scanner can only interpret the answers by reading the darkened responses in the manner explained in preceding paragraph. Darkening more than one circle/oval as response to a question shall also be considered as wrong answer.</p>	<p>15. <b>There is no negative marking</b> for wrong answers or questions not attempted by the candidate.</p>

1. Which of the following is true for 'Sound'?
  - (a) Sound cannot travel through vacuum
  - (b) Sound cannot travel through gas
  - (c) Sound cannot travel through liquids
  - (d) Sound cannot travel through solids
  
2. Oxidation is a chemical reaction involving the:
  - (a) Gain of Electrons
  - (b) Loss of Electrons
  - (c) Gain of Protons
  - (d) Loss of Protons
  
3. The pancreas secretes:
  - (a) Bile
  - (b) Peurine
  - (c) Insulin
  - (d) None of the above
  
4. Which of the following is not an example of environmental carcinogen?
  - (a) Arsenic
  - (b) Formaldehyde
  - (c) Asbestos
  - (d) Carbon
  
5. Which one of the following is the oldest Veda?
  - (a) Yajur Veda
  - (b) Rig Veda
  - (c) Sam Veda
  - (d) Atharva Veda
  
6. Who wrote Indica?
  - (a) Alexander
  - (b) Megasthenes
  - (c) Selucus
  - (d) Huen Tsang
  
7. Which one of the following was one of the main causes of the Sepoy Mutiny or the Indian Rebellion of 1857?
  - (a) Jawan-Officer rivalry
  - (b) Pay disputes
  - (c) Non-functional weapons
  - (d) Greased cartridges
  
8. Hitler : Germany :: Mussolini : ?
  - (a) Italy
  - (b) Poland
  - (c) Hungary
  - (d) Austria

9. Which of the following statements are correct with regard to NHRC?
1. NHRC is neither a constitutional body nor a statutory body.
  2. After their tenure ends the Chairman and members are not eligible for further employment under the central or state government.
  3. Its recommendations are binding on the concerned government or authority.
- (a) 1 & 2 only  
(b) 1 & 3 only  
(c) 1, 2 & 3  
(d) 2 only
10. A session of the Parliament can only be summoned by:
- (a) The Prime Minister  
(b) The President  
(c) The Speaker of the Lok Sabha  
(d) None of the above
11. The power to increase the number of judges in the Supreme Court of India is vested in:
- (a) The Law Commission  
(b) The Chief Justice of India  
(c) The President  
(d) The Parliament
12. The power of the Supreme Court of India to decide disputes between the Centre and the States falls under its:
- (a) Advisory Jurisdiction  
(b) Appellate Jurisdiction  
(c) Original Jurisdiction  
(d) Discretionary Jurisdiction
13. The US has agreed to India becoming a member of MTCR, which stands for:
- (a) Manufacturing Technology Cooperation Regime  
(b) Monsoon Technology Coordination Regime  
(c) Medium Term Cooperation Regime  
(d) Missile Technology Control Regime
14. Which country recently voted on a referendum on exit from the European Union?
- (a) Greece  
(b) Poland  
(c) Britain  
(d) Ukraine
15. In India, the term 'hot money' is used to refer to:
- (a) Currency + Reserves with the RBI  
(b) Net GDR  
(c) Net Foreign Direct Investment  
(d) Foreign Portfolio Investment

16. Which one of the following air pollutants combines with the haemoglobin of human blood and reduces its oxygen-carrying capacity, leading to suffocation and may cause even death?
- (a) Chlorofluorocarbon
  - (b) Fly ash
  - (c) Carbon monoxide
  - (d) Sulphur dioxide

17. Neil O'Brien, who died recently, was a famous:

- (a) Journalist
- (b) Billiards player
- (c) Dramatist
- (d) Quiz master

18. Recently, the Government of India cleared the proposal for the production of 18 indigenous 'Dhanush' artillery guns to be produced in India by:

- (a) Indian Army
- (b) US Army
- (c) Indian Ordnance Factory Board
- (d) Indian and US Army jointly

19. Which of the following statements concerning atmosphere of the Earth are correct?

1. In stratosphere, temperature increases with altitude
2. In mesosphere, temperature decreases with altitude
3. The lowest temperature of the atmosphere is recorded in the upper part of mesosphere
4. Tropopause is an isothermal zone

Select the correct answer using the code given below:

- (a) 1 and 2 only
- (b) 1, 2 and 3 only
- (c) 3 and 4 only
- (d) 1, 2, 3 and 4

20. Which of the following statements regarding soil is/are correct?

1. Alluvial soils are rich in nitrogen content
2. Black soils are rich in iron and lime but deficient in nitrogen
3. Laterite soil are rich in iron and aluminium but deficient in nitrogen and potassium

Select the correct answer using the code given below:

- (a) 1 and 2 only
- (b) 3 only
- (c) 2 and 3 only
- (d) 1, 2 and 3

21. Which one of the following was the earlier name of Tokyo?

- (a) Osaka
- (b) Kyoto
- (c) Samurai
- (d) Edo

22. Consider the following statements regarding the National Green Tribunal (NGT):
1. NGT can handle cases related to The Public Liability Insurance Act, 1991
  2. It is a federal legislation enacted by the Parliament of India, under India's constitutional provision of Article 21

Which of the statements given above is/are incorrect?

- (a) Only 1
- (b) Only 2
- (c) Both 1 and 2
- (d) Neither 1 nor 2

23. Consider the following statements based on the UN report on e-waste:

1. The bulk of global e-waste constitutes from mobile phones, calculators, personal computers, printers and small information technology equipment
2. The lowest amount of e-waste per inhabitant was generated in the South-Asian countries

Which of the statements given above is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

24. Heinz Award is given in the fields of

1. Art
2. Environment
3. Public Policy
4. Mathematics
5. Technology

Select the correct answer using the codes given below:

- (a) 1, 2, 3, 4 and 5
- (b) 2, 3, 4 and 5
- (c) 1, 2, 3 and 5
- (d) 1, 2, 4 and 5

25. Apart from vehicles, the Bharat Stage (BS) norms are also the emission standards for

1. Industrial chimneys
2. Mining corporations
3. Natural gas exploration companies

Select the correct answer using the codes given below:

- (a) 1 and 2
- (b) 2 only
- (c) 3 only
- (d) None of the above

26. Which of the following statements about the NALSA is incorrect?

- (a) It is a constitutional body
- (b) It was established to achieve the objectives of Article 39A
- (c) It organizes Lok Adalats for the settlement of disputes
- (d) It was instrumental in getting the third gender status for transgender

27. If the numerator of a fraction is increased by 200% and the denominator of the fraction is increased by 120%, the resultant fraction is  $\frac{4}{11}$ . What is the original fraction?

- (a)  $\frac{4}{15}$
- (b)  $\frac{3}{11}$
- (c)  $\frac{5}{12}$
- (d)  $\frac{6}{11}$

28. Ninad, Vikas and Manav enter into a partnership. Ninad invests some amount at the beginning. Vikas invests double the amount after 6 months and Manav invests thrice the amount invested by Ninad after 8 months. They earn a profit of Rs. 45,000 at the end of the year. What is Manav's share in the profit?

- (a) Rs. 25,000
- (b) Rs. 15,000
- (c) Rs. 12,000
- (d) Rs. 9,000

29. In a certain code SPORADIC is written as QNORDJEB. How is TROUBLES written in that code?

- (a) SQTNTFMC
- (b) TNQSRDKA
- (c) TNQSTFMC
- (d) TFQSCMFT

30. Statements:

- All stations are houses.
- No house is garden.
- Some gardens are rivers.
- All rivers are ponds.

Conclusions:

- I. Some ponds are gardens.
  - II. Some ponds are stations.
  - III. Some ponds are houses.
  - IV. No pond is station.
- (a) Only I follows
  - (b) Only either II or IV follows
  - (c) Only I and IV follow
  - (d) None of these

31. The value of the Dirichlet integral  $\iiint_T x^3 y^3 z^3 dx dy dz$ , where T is the region in the first octant

bounded by the sphere,  $x^2 + y^2 + z^2 = 1$  and the co-ordinate planes is

- (a)  $\frac{1}{5760}$
- (b)  $\frac{1}{720}$
- (c)  $\frac{1}{1440}$
- (d)  $\frac{1}{2240}$

32. Consider the LPP:

Maximize  $Z = 3x_1 + 2x_2 + 5x_3$  subject to

$$x_1 + 2x_2 + x_3 + x_4 = 8, 3x_1 + 2x_3 + x_5 = 12, x_1 + 4x_2 + x_6 = 8, x_i \geq 0 \forall i$$

If  $(x_2, x_3, x_6)$  forms the optimal basis, then the optimal value of the dual problem is

- (a) 30
- (b) 35
- (c) 40
- (d) 45

33. The joint probability density function (p.d.f.) of two random variables  $X$  and  $Y$  is

$$f(x + y) = \begin{cases} k[(x + y) - (x^2 + y^2)]; & 0 < x < 1, 0 < y < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Then the correlation coefficient between  $X$  and  $Y$  is

- (a) 0
- (b) 1/2
- (c) -1/2
- (d) 1/4

34. Consider the following statements

- (i) Every compact subset of a metric space is closed
- (ii) Closed subsets of compact sets are compact

Then

- (a) Only (i) is true
- (b) Both (i) and (ii) are true
- (c) Both (i) and (ii) are false
- (d) Only (ii) is true

35. Let the LPP be:

Maximize  $Z = 3x_1 + 2x_2 + 5x_3$  subject to

$$x_1 + 2x_2 + x_3 + x_4 = 8, 3x_1 + 2x_3 + x_5 = 12, x_1 + 4x_2 + x_6 = 8, x_i \geq 0 \forall i. \text{ If } (x_2, x_3, x_6)$$

forms the optimal basis, and inverse of the optimal basis is

$$B^{-1} = \begin{pmatrix} 0.5 & x & 0 \\ y & 0.5 & 0 \\ -2 & 1 & z \end{pmatrix}. \text{ Then } x + y + z \text{ is}$$

- (a) -1
- (b) 0.0
- (c) 0.5
- (d) 0.75

36. Let  $p$  be the probability that a man aged  $x$  years will die in a year. Then the probability that out of  $n$  men  $A_1, A_2, \dots, A_n$  each aged  $x$  years,  $A_1$  will die in a year and will be the first to die is

- (a)  $\frac{1}{n}[1 - p^n]$
- (b)  $\frac{1}{n}[1 - (1 + p)^n]$
- (c)  $\frac{1}{n}[1 + (1 - p)^n]$
- (d)  $\frac{1}{n}[1 - (1 - p)^n]$

37. Let the joint probability density function of two random variables  $X$  and  $Y$  be

$$f(x+y) = \begin{cases} (6-x-y)/8; & 0 \leq x \leq 2, 2 \leq y \leq 4, \\ 0 & \text{otherwise.} \end{cases}$$

Then the correlation coefficient between  $X$  and  $Y$  is

- (a) 11/36
- (b) 1/11
- (c) -1/11
- (d) 2/11

38. Let  $h$  be step size that can be used in the tabulation of function  $f(x)$ ,  $a \leq x \leq b$  at equally spaced nodal points. If  $|f^{(iv)}(x)| \leq M$  for  $x \in [a, b]$ , then the upper bound of truncation error of the cubic interpolation is

- (a)  $\frac{h^4 M}{24}$
- (b)  $\frac{h^2 M}{8}$
- (c)  $\frac{h^3 M}{4!}$
- (d)  $\frac{h^3 M}{3!}$

39. Let  $T_1$  and  $T_2$  be any two topologies on  $X$ . Then

- (a)  $T_1 \cup T_2$  is also a topology on  $X$
- (b)  $T_1 + T_2$  is also a topology on  $X$
- (c)  $T_1 \cap T_2$  is also a topology on  $X$
- (d)  $T_1 T_2$  is also a topology on  $X$

40. Time period for a simple pendulum of length  $l$  is

- (a)  $\pi \sqrt{\frac{l}{g}}$
- (b)  $2\pi \sqrt{\frac{g}{l}}$
- (c)  $2\pi \sqrt{\frac{l}{g}}$
- (d)  $\pi \sqrt{\frac{g}{l}}$



41. For each  $a \in R$  consider the LPP

Maximize  $z = x_1 + 2x_2 + 3x_3 + 4x_4$  subject to

$$ax_1 + 2x_3 \leq 1, x_1 + ax_2 + 3x_4 \leq 2, x_1, x_2, x_3, x_4 \geq 0.$$

Let  $S = \{a \in R : \text{the given LPP has a basic feasible solution}\}$ .

(a)  $S = \phi$

(b)  $S = R$

(c)  $S = (0, \infty)$

(d)  $S = (-\infty, 0)$

42. Consider the LPP:

Maximize  $Z = 3x_1 + 2x_2 + 5x_3$ , subject to

$$x_1 + 2x_2 + x_3 + x_4 = 430, 3x_1 + 2x_3 + x_5 = 460, x_1 + 4x_2 + x_6 = 420, x_i \geq 0 \forall i.$$

Let  $(x_2, x_3, x_6)$  form the optimal basis and inverse of the optimal basis be

$$B^{-1} = \begin{pmatrix} 0.5 & \alpha & 0 \\ \beta & 0.5 & 0 \\ -2 & 1 & 1 \end{pmatrix}. \text{ Then } \alpha + \beta \text{ is}$$

(a) -0.25

(b) -0.5

(c) 0

(d) 1

43. The general solution of the partial differential equation  $(y - z) \frac{\partial z}{\partial x} + (z - x) \frac{\partial z}{\partial y} = x - y$  is

(a)  $\Phi(x + y + z, x^2 + y^2 + z^2) = 0$

(b)  $\Phi(xyz, x + y + z) = 0$

(c)  $\Phi(xyz, x^2 + y^2 + z^2) = 0$

(d)  $\Phi(x^2 - y^2 - z^2, x - y - z) = 0$

44. Consider the integral equation

$$y(x) = 3x^2 + \int_0^x y(t) \sin(x-t) dt, x \in [0, \pi].$$

Then, the value of  $y(2)$  is

(a) 12

(b) 14

(c) 16

(d) 18

45. The curve on which the functional

$$I(x, y, y') = \int_0^1 ((y')^2 + 12xy) dx$$

with  $y(0) = 0$  and  $y(1) = 1$  can be extremized is

- (a)  $y = x^2$
- (b)  $y = x^3$
- (c)  $y = x^4$
- (d)  $y = x + x^2$

46. If the independent random variables  $X$  and  $Y$  are binomially distributed with parameters  $n = 3, p = 1/3$  and  $n = 5, p = 1/3$  respectively. Then  $P(X + Y \geq 1)$  is

- (a)  $\frac{2}{3}$
- (b)  $\left(\frac{2}{3}\right)^8$
- (c)  $1 - \left(\frac{2}{3}\right)^8$
- (d)  $\left(\frac{2}{3}\right)^7$

47. The value of the integral  $\int_0^\infty e^{-(t-3)} \delta(t-3) dt$ ; where  $\delta(t)$  denotes dirac delta function, is

- (a)  $e^3$
- (b)  $e^{-3}$
- (c) 1
- (d)  $\infty$

48. The point lying on the solution curve of the differential equation  $yy' + x = 0$  with the condition  $y(1) = -1$  is

- (a) (1,1)
- (b)  $(\sqrt{2}, \sqrt{2})$
- (c)  $(1, \sqrt{2})$
- (d)  $(\sqrt{2}, 1)$

49. Let  $f(x) = \sin x, 0 \leq x \leq \frac{\pi}{2}$  and let  $P = \{0, \frac{\pi}{2n}, \frac{2\pi}{2n}, \dots, \frac{n\pi}{2n}, \}$  be a partition of  $[0, \frac{\pi}{2}]$ .

Then lower Riemann sum is

- (a) 1
- (b) 2
- (c) 3
- (d) 10

50.  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y}{x^6 + y^2}$

- (a) is 0
- (b) is 1
- (c) is  $\infty$
- (d) does not exist

51. Which of the following is not true

- (a) The function space  $C[a, b]$  with norm  $\|x\|_{\infty} = \max_{t \in [a,b]} |x(t)|$  is not complete
- (b)  $C[a, b]$  is a complete normed linear space
- (c)  $\|\cdot\|_{\infty}$  is called the sup norm (or uniform norm on  $C[a, b]$ )
- (d) None of these

52. Consider the LPP:

Maximize  $Z = 3x_1 + 2x_2 + 5x_3$  subject to

$$x_1 + 2x_2 + x_3 + x_4 = 8, 3x_1 + 2x_3 + x_5 = 12, x_1 + 4x_2 + x_6 = 8, x_i \geq 0 \forall i$$

If  $(x_2, x_3, x_6)$  form the optimal basis, and the inverse of the optimal basis be

$$B^{-1} = \begin{pmatrix} 0.5 & -0.25 & 0 \\ 0 & 0.5 & 0 \\ -2 & 1 & 1 \end{pmatrix}. \text{ If the column of } x_1 \text{ in the optimal table is } [-0.25, \alpha, 2]^T, \text{ then the}$$

value of  $\alpha$  is

- (a) 1.0
- (b) 1.5
- (c) 2.0
- (d) 2.5

53. The general solution of  $x(y^2 - z^2) \frac{\partial z}{\partial x} + y(z^2 - x^2) \frac{\partial z}{\partial y} = z(x^2 - y^2)$  is

- (a)  $\Phi(x + y + z, x^2 + y^2 + z^2) = 0$
- (b)  $\Phi(x + y + z, xyz) = 0$
- (c)  $\Phi(xyz, x^2 + y^2 + z^2) = 0$
- (d)  $\Phi(x^2 y^2 z^2, x^2 + y^2 + z^2) = 0$

54. The solution of the integral equation  $y(x) = x + \frac{1}{6} \int_0^x (x-t)^3 y(t) dt$  is

(a)  $y(x) = \frac{1}{2}(\sin x + \cos x)$

(b)  $y(x) = \frac{1}{2}(\sin x + \sinh x)$

(c)  $y(x) = \frac{1}{2}(\sinh x + \cos x)$

(d)  $y(x) = \frac{1}{2}(\cos x + \cosh x)$

55. The extremal of the functional  $F(x, y, y') = \int_{x_0}^{x_1} (1 + x^2 y') y' dx$  is

(a)  $y = c_1 x^2 + c_2$

(b)  $y = \frac{c_1}{x^2} + c_2$

(c)  $y = \frac{c_1}{x} + c_2$

(d)  $y = c_1 x^3 + c_2$

56. Let the random variable  $X$  have normal distribution with mean 12 and standard deviation 4. Then  $P(0 \leq X \leq 12)$  is

(a) 0.4986

(b) 0.9772

(c) 0.2612

(d) 0.3112

57. A valid solution for a fourth degree linear homogeneous ordinary differential equation with constant coefficient is

(a)  $(c_1 + c_2 x) \cos x + (c_3 + c_4 x) \sin x$

(b)  $c_1 \cos x + c_2 \sin x + c_3(\cos x + \sin x) + c_4(\cos x - \sin x)$

(c)  $c_1 + c_2 x + c_3 \sin x + c_4 \cos x \sin x$

(d)  $(c_1 + c_2 x) \cos x + (c_3 + c_4 x) \cos x \sin x$

58. Consider an iterative scheme  $\epsilon_{n+1} = C\epsilon_n^3 + \epsilon_n^4 + O(\epsilon_n^5)$ , where  $\epsilon_n = x_k - \xi$  is the error in the  $n^{\text{th}}$  iteration. Then the rate of convergence of this scheme is

(a) 4

(b) 3

(c) 2

(d) 1

59. Let  $Ax = b$  be a linear system of  $n$  unknowns. Consider the following two statements

- (I)  $A$  is strictly diagonally dominant matrix.
- (II) Gauss-Jacobi method converges for any initial starting vector.

Then

- (a) (I) implies (II)
- (b) (II) implies (I)
- (c) Both (I) and (II) imply each other
- (d) All of the above

60. Let  $\varphi: \mathbb{Z}(\sqrt{2}) \rightarrow \mathbb{Z}(\sqrt{2})$  be defined as  $\varphi(a+b\sqrt{2}) = a-b\sqrt{2}$  for each  $a, b \in \mathbb{Z}$ . Then which of the following is not true?

- (a)  $\varphi$  is a ring homomorphism
- (b)  $\varphi$  is onto
- (c)  $\text{Kernel}(\varphi) \neq \{0\}$
- (d)  $\mathbb{Z}(\sqrt{2})$  is not a ring

61. Consider the following three norms for  $x = (x_1, x_2, \dots, x_n) \in \mathfrak{R}^n$

$$(i) \|x\|_2 = \left( \sum_{i=1}^n |x_i|^2 \right)^{1/2} \quad (ii) \|x\|_1 = \sum_{i=1}^n |x_i| \quad (iii) \|x\|_\infty = \max\{|x_i| : 1 \leq i \leq n\}$$

Then  $\mathfrak{R}^n$  is Banach space with the norm defined

- (a) only on (i) and (ii)
- (b) only on (ii) and (iii)
- (c) only on (i) and (iii)
- (d) on (i), (ii) and (iii)

62. Let  $\|\cdot\|$  and  $\|\cdot\|'$  be equivalent norms on a linear space  $X$ . Then which of the following is not true?

- (a)  $(X, \|\cdot\|)$  is a Banach space implies that  $(X, \|\cdot\|')$  is a Banach space
- (b)  $(X, \|\cdot\|')$  is a Banach space implies that  $(X, \|\cdot\|)$  is a Banach space
- (c) A set is bounded in  $(X, \|\cdot\|)$  implies that it is bounded in  $(X, \|\cdot\|')$
- (d) The Cauchy sequences in both the normed spaces may be different

63. Let  $Y$  be a subspace of a Hilbert space  $H$ . Then which of the following is not true?

- (a)  $Y$  is closed implies that  $Y$  is not a Hilbert space
- (b)  $Y$  is closed implies that  $Y$  is complete
- (c)  $Y$  is complete implies that  $Y$  is closed
- (d)  $\overline{Y}$  is closed

64. Let  $A=(0, 1)$ ,  $B=(1, 2)$  and  $C=[2, 3)$ . Then

- (a)  $A$  and  $B$  are not separated
- (b)  $B$  and  $C$  are not separated
- (c)  $B$  and  $C$  are separated
- (d) All of the above

65. Which of the following is not true?

- (a) Every metric space is Hausdorff
- (b) Every metric space is a  $T_1$  – space
- (c) Limit of a sequence in a Hausdorff space is unique
- (d) A metric space may not be a Hausdorff

66. Consider the following two statements

- (I) Kinetic energy can be represented as a homogeneous function of generalized coordinates and generalized velocities in a conservative system.
- (II) Potential energy does not depend upon generalized velocities

Then

- (a) Only (I) is correct
- (b) Only (II) is correct
- (c) Both (I) and (II) are correct
- (d) Both (I) and (II) are not correct

67. Let the optimal solution of an LPP occur at two vertices  $X_1$  and  $X_2$ . Then  $X = \alpha_1 X_1 + \alpha_2 X_2$ ,  $\alpha_1 + \alpha_2 = 1$ ,  $\alpha_1, \alpha_2 > 0$  is

- (a) not optimal solution but a basic solution
- (b) an optimal BFS
- (c) not a basic solution but optimal solution
- (d) neither optimal solution nor basic solution

68. The general solution of the partial differential equation  $xz \frac{\partial z}{\partial x} + yz \frac{\partial z}{\partial y} = xy$  is

(a)  $\Phi\left(xy - z, \frac{x}{z}\right) = 0$

(b)  $\Phi(xy - z^2, \frac{x}{y}) = 0$

(c)  $\Phi\left(xy + z, \frac{x^2}{y}\right) = 0$

(d)  $\Phi\left(xyz, \frac{x}{y}\right) = 0$

69. Let the integral equation be  $y(x) = x^2 + \int_0^x y(t) \sin(x-t) dt, x \in [0, 2]$ . Then the value of  $y(2)$  is

(a)  $\frac{19}{3}$

(b)  $\frac{20}{3}$

(c)  $\frac{22}{3}$

(d)  $\frac{23}{3}$

70. Let  $P_2(\mathbb{R})$  be the vector space of polynomials of degree at most 2, and  $B$  be its one of the bases, where  $B = \{1, 1 + x, 1 + x^2\}$ . The coordinate vector  $[P]_B$  of the polynomial  $p(x) = 1 - x^2$  is

(a)  $[1 \ 2 \ 3]^T$

(b)  $[3 \ 2 \ 1]^T$

(c)  $[2 \ -2 \ 1]^T$

(d)  $[1 \ 1 \ -1]^T$

71. The image of the half plane  $Im(z) > 1$  under the transformation  $w = (1 - i)z + 1 - 2i$  is

(a) the half plane  $v > (1 - u)$

(b) the half plane  $v > (1 + u)$

(c) the half plane  $v < (1 - u)$

(d) the half plane  $v < (1 + u)$

72. The order of element  $(6\mathbb{Z}+5)$  in the quotient group  $\mathbb{Z}/6\mathbb{Z}$  is

(a) 5

(b) 6

(c) 30

(d) infinite

73. The Fourier series expansion of the periodic function

$f(x) = x, -\pi \leq x \leq \pi, f(x+2\pi) = f(x)$  is

(a)  $x = 2\left[\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} + \dots\right]$

(b)  $x = \left[\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} + \dots\right]$

(c)  $x = \left[\sin x + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \frac{\sin 4x}{4} + \dots\right]$

(d)  $x = 2\left[\sin x + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \frac{\sin 4x}{4} + \dots\right]$

74. Consider the LPP:

Maximize  $Z = ax_1 + 6x_2, a \in R$

subject to  $4x_1 + 3x_2 \leq 12, 3x_1 + 4x_2 \leq 12, x_1, x_2 \geq 0$ . Then,

(a) the primal has an optimal solution but the dual does not have an optimal solution

(b) both primal and dual has an optimal solutions

(c) the dual has an optimal solution but the primal does not have an optimal solution

(d) neither the primal nor the dual have optimal solutions

75. The general solution of the partial differential equation  $(x+y)\frac{\partial z}{\partial x} - (x+y)\frac{\partial z}{\partial y} = z$  is

(a)  $\Phi(xy, x+y+\log z) = 0$

(b)  $\Phi((x+y)\log z, x+y) = 0$

(c)  $\Phi(x+y, x-(x+y)\log z) = 0$

(d)  $\Phi(x-y(x+y)\log z, x+y) = 0$

76. Consider a linear transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  such that  $T(3,1) = (4,2)$  and  $T(-1,3) = (2,-4)$ . Then  $T(3,6)$  is

(a)  $(9,3)$

(b)  $(9,-3)$

(c)  $(-9,3)$

(d)  $(-9,-3)$

77. Let  $A$  be a square matrix of order 4 with  $\text{rank}(A) = 2$  and  $\text{rank}(A+I) = 2$ . Then eigenvalues of  $A$  are

(a)  $1,1,0,0$

(b)  $1,0,0,0$

(c)  $1,1,1,0$

(d)  $1,1,1,1$



78. A man alternatively tosses a coin and throws a die, beginning with coin. The probability that he will get a head before he gets a '5' or '6' on die is

- (a)  $\frac{1}{4}$
- (b)  $\frac{1}{2}$
- (c)  $\frac{3}{4}$
- (d)  $\frac{1}{3}$

79. The function  $w = (i + 1)z$  maps the right half plane  $Re(z) > 1$  onto

- (a) upper half plane  $Im(w) > 2$
- (b) lower half plane  $Im(w) < 2$
- (c) right half plane  $Re(w) > 1$
- (d) right half plane  $Re(w) > 2$

80. Let  $c: z(t) = \cos t + i \sin t$  for  $0 \leq t \leq \frac{\pi}{2}$ , Then the integral  $\int_c (z + 1) dz$  is

- (a) 2
- (b)  $i + 2$
- (c)  $2i - 1$
- (d)  $i - 2$

81. Let  $h$  be step size that can be used in the tabulation of function  $f(x)$ ,  $a \leq x \leq b$  at equally spaced nodal points and the truncation error of the quadratic interpolation is less than  $\mathcal{E}$ . If  $|f'''(x)| \leq M_3$  for  $x \in [a, b]$ , then step size  $h$  is less than

- (a)  $\left(\frac{24\mathcal{E}}{M_3}\right)^{\frac{1}{4}}$
- (b)  $\left(\frac{9\sqrt{3}\mathcal{E}}{M_3}\right)^{\frac{1}{3}}$
- (c)  $\left(\frac{8\mathcal{E}}{M_3}\right)^{\frac{1}{2}}$
- (d)  $\left(\frac{4\mathcal{E}}{M_3}\right)^{\frac{1}{2}}$

82. The number of elements in the field  $\mathbb{Z}_{31}[x]/\langle x^3-9 \rangle$  is

- (a) 31
- (b)  $31^2$
- (c)  $31^3$
- (d)  $31^4$

83. Consider the series  $\sum \frac{(-1)^{n-1}}{n+x^2}$ . Then

- (a) the series converges uniformly for all real values of  $x$
- (b) the series converges uniformly only for  $x \in (0, \infty)$
- (c) the series converges uniformly only for  $x \in [0, 1]$
- (d) the series converges uniformly only for  $x \in (-\infty, 0)$

84. Consider the LPP:

Maximize  $Z = 3x_1 + 2x_2 + 5x_3$  subject to

$$x_1 + 2x_2 + x_3 + x_4 = 8, \quad 3x_1 + 2x_3 + x_5 = 12, \quad x_1 + 4x_2 + x_6 = 8, \quad x_i \geq 0 \quad \forall i.$$

If  $(x_2, x_3, x_6)$  forms the optimal basis, and inverse of the optimal basis is

$$B^{-1} = \begin{pmatrix} 0.5 & -0.25 & 0 \\ 0 & 0.5 & 0 \\ -2 & 1 & 1 \end{pmatrix}, \text{ then the value of } \alpha \text{ for which the column of } x_4 \text{ in the optimal}$$

table is  $[0.5, \alpha, -2]^T$  is

- (a) -1
- (b) 0.0
- (c) 0.5
- (d) 1.0

85. The general solution of  $x^2(y-z)\frac{\partial z}{\partial x} + y^2(z-x)\frac{\partial z}{\partial y} = z^2(x-y)$  is

- (a)  $\Phi\left(xyz, \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$
- (b)  $\Phi(xyz, x+y+z) = 0$
- (c)  $\Phi\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}, x+y+z\right) = 0$
- (d)  $\Phi\left(x^2 + y^2 + z^2, \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$

86. The complete integral of  $p = e^q$  is

- (a)  $z = e^a x + 2e^a y + c$ , where  $a$  and  $c$  are arbitrary constants
- (b)  $z = ax + e^a y + c$ , where  $a$  and  $c$  are arbitrary constants
- (c)  $z = ax + (\log a)y + c$ , where  $a$  and  $c$  are arbitrary constants
- (d)  $z = (\log a)x + 2e^a y + c$ , where  $a$  and  $c$  are arbitrary constants

87. Consider the integral equation

$$y(x) = x^2 + \int_0^x y(t) \sin(x-t) dt, x \in [0, 2].$$

Then the value of  $y(1)$  is

- (a)  $\frac{7}{12}$
- (b)  $\frac{11}{12}$
- (c)  $\frac{13}{12}$
- (d)  $\frac{15}{12}$

88. If 1 and 3 are the eigenvalues of the matrix  $\begin{bmatrix} 5 & 4 \\ x & -1 \end{bmatrix}$ . Then  $x$  is

- (a) -2
- (b) -3
- (c) 2
- (d) 3

89. If the eigenvalues of  $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$  are  $\lambda_1, \lambda_2$  and  $\lambda_3$ , then

- (a)  $|\lambda_1| + |\lambda_2| + |\lambda_3| = 3$
- (b)  $|\lambda_1| + |\lambda_2| - |\lambda_3| = 3$
- (c)  $|\lambda_1| - |\lambda_2| + |\lambda_3| = 3$
- (d)  $-|\lambda_1| + |\lambda_2| + |\lambda_3| = 3$

90. A perfect cube is thrown a large number of times in sets of 8. The occurrence of a 2 or 4 is called a success. Then in what proportion of the sets, 3 successes can be expected?

- (a) 21.39 %
- (b) 24.37 %
- (c) 27.31 %
- (d) 29.12 %

91. The singularity of  $(\sin(1/z))^{-1}$  is

- (a) Simple poles at  $z = \frac{1}{n\pi}$  for  $n = \pm 1, \pm 2, \dots$
- (b) A non-isolated singularity at the origin
- (c) Essential singularity at  $z = 0$
- (d) Both (a) and (b)

92. Let  $c$  be an upper semicircle with radius 1 centered at  $x = 2$  and oriented in counter-clockwise direction. Then value of the complex integral  $\int_c \frac{1}{(z-2)} dz$  is

- (a) 0
- (b) 1
- (c)  $i\pi$
- (d)  $\pi$

93. The Laplace transform of  $f(t) = \begin{cases} 0 & 0 \leq t < 3 \\ (t-3)^2 & t \geq 3 \end{cases}$  is

(a)  $\frac{2}{s^3} e^{-3s}$

(b)  $\frac{3}{s^3} e^{-3s}$

(c)  $\frac{1}{s^3} e^{-3s}$

(d)  $e^{-3s}$

94. Let  $y(x)$  be the solution of the differential equation  $4y'' + 12y' + 9y = 0$ ;  $y(0) = 1, y'(0) = -4$ . Then  $y(1)$  is

(a)  $-\frac{1}{2} e^{-3/2}$

(b)  $-\frac{3}{2} e^{-3/2}$

(c)  $-\frac{5}{2} e^{-3/2}$

(d)  $-\frac{7}{2} e^{-3/2}$

95. Consider that  $|f''(x)| \leq M$  for  $x \in [a, b]$  and  $h$  is step size that can be used in the tabulation of function  $f(x)$ ,  $a \leq x \leq b$  at equally spaced nodal points. The upper bound of truncation error of the linear interpolation is

(a)  $\frac{hM}{8}$

(b)  $\frac{h^2M}{8}$

(c)  $\frac{h^2M}{6}$

(d)  $\frac{hM}{6}$

96. The Gauss Siedel method converges if the spectral radius of the iteration matrix is

(a) any finite value

(b) any positive real value

(c) any negative real value

(d) in the interval  $(-1, 1)$

97. The number of generators of a cyclic group of order 35 is

(a) 5

(b) 7

(c) 18

(d) 24

98. If  $-1 < \eta < 1$ , then error in three-point formula of the Gauss-Quadrature rule is

(a)  $\frac{f^{(vi)}(\eta)}{15750}$

(b)  $\frac{f^{(vi)}(\eta)}{13570}$

(c)  $\frac{f^{(vi)}(\eta)}{17570}$

(d)  $\frac{f^{(vi)}(\eta)}{19570}$

99. Let  $AX = b$  be a system of linear equations. If  $A=L+D+U$ , where L, D, U are lower triangular, diagonal and upper triangular matrices respectively, then the Jacobi iterative scheme for the solution of the system is

(a)  $X^{(k+1)} = X^{(k)} - D^{-1}(-D+L+U)X^{(k)} + D^{-1}b$

(b)  $X^{(k+1)} = X^{(k)} - D^{-1}(D+L+U)X^{(k)} + D^{-1}b$

(c)  $X^{(k+1)} = X^{(k)} - D^{-1}(D-L+U)X^{(k)} + D^{-1}b$

(d)  $X^{(k+1)} = X^{(k)} - D^{-1}(D+L-U)X^{(k)} + D^{-1}b$

100. Which of the following is not metrizable?

(a) Discrete space

(b) Real line R with usual topology

(c) Indiscrete space

(d) Housdroff space

\*\*\*\*\*