CSM – 53 / 15
Mathematics
Paper – II

Time : 3 hours

Full Marks : 300

The figures in the right-hand margin indicate marks.

*Candidates should attempt Q. No. 1 from Section – A and Q. No. 5 from Section – B which are compulsory and **three** of the remaining questions, selecting at least **one** from each Section.*

Section – A

1. Answer any **five** of the following : $12 \times 5 = 60$

   (a) Find the lowest possible degree polynomial which assume the value 12, 3, –21, 15 when x has the values 2, 3, –1, 1. Hence, find the value of the function when x = 0.

   (b) Find by Newton-Raphson method the real root of $3x - \cos x - 1 = 0$, correct upto five decimal places.

(c) Define connected graph. If G is connected graph with $n(\geq 3)$ vertices then at least one vertex must have degree > 1.

(d) Let $G = (V, E)$ be a graph. Let $\chi(G)$ = chromatic number of G and $\Delta(G) = \max_{v_i \in V(G)} d_G(v_i)$. Then $\chi(G) = \leq 1 + \Delta(G)$.

(e) Solve $(D^2 + a^2)y = \sec ax$.

(f) Solve $(D^2 + 2DD' + D'^2)z = e^{2x+3y}$.

2. (a) Find y(1) by Euler's methods from the differential equation $\frac{dy}{dx} = -\frac{y}{1+x}$ when $y(0.3) = 2$, correct upto four decimal places, taking step length h.= 0.1. 20

(b) Compute Simpson one-third rule, the integral $\int_0^1 x^2(1-x)\,dx$ correct upto three decimal places, step length is 0.1. 20

(c) Determine the least square approximation of the type $ax^2 + bx + c$ to the function 2^x at the points $x_i = 0, 1, 2, 3, 4$. 20

3. (a) $G = (V, E)$ be a graph with the vertices ordered as $V = \{v_1, v_2, \dots\dots\dots v_n\}$ and let A_G denotes the adjacency matrix of G. Then prove that the number of walks of length k between v_i and v_j is the (i, j)th entry of A_G^k.

20

(b) If G is a connected graph with n vertices then prove that G has at least n – 1 edge.

20

(c) If a tree has a vertex of degree k then prove that it has at least k pendant vertices. 20

4. (a) Solve $(2xz - yz)\,dx + (2yz - zx)\,dy - (x^2 - xy + y^2)\,dz = 0$. 20

(b) Using Charpit's method, find the complete integral of the equation $2(z + xp + yq) = yp^2$.

20

(c) Solve the differential equation of Laplace transformation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-t}$, given $y(0) = y'(0) = 0$. 20

Section – B

5. Answer any **three** of the following :

(a) Forces P, Q, R act along the sides of a triangle formed by the lines $x + y - 1 = 0$, $x - y + 1 = 0$, $y = 2$. Find the magnitude and line of action of the resultant. 20

(b) Develop a flow chart to calculate the roots of $ax^2 + bx + c = 0$ for various values of a, b, c. 20

(c) Find the graphical solution of the following LPP : 20

Maximize $Z = 2x_1 - x_2$

Subject to $x_1 - x_2 \leq 1$

$x_1 \leq 3$

$x_1 \geq 0, x_2 \geq 0.$

(d) A velocity field is given by $q = -xi + (y + t)j$. Find the stream function and the streamlines for this field for $t = 2$. 20

6. (a) Determine whether the motion specified by $q = A(xj - iy) / (x^2 + y^2)$, (A is a constant) is a possible motion for an incompressible fluid.

If so, determine the equation of streamlines. Also, show that the motion is of potential kind. Find the velocity potential. 30

(b) A source S and a sink T of equal strengths m are situated within the space bounded by a circle whose centre is O. If S and T are equal distances from O on opposite sides of it and on the same diameter AOB, show that the velocity of the liquid at any point P is

$$2m\frac{OS^2+OA^2}{OS}\cdot\frac{PA\cdot PB}{PS\cdot PS'\cdot PT\cdot PT'},$$

where S′ and T′ are the inverse points of S and T with regard to the circle. 30

7. (a) Find the moment of inertia of a uniform rectangular lamina about a line through its centre and perpendicular to its plane. 30

(b) A semicircular disc rests in a vertical plane with its curved edge on a rough horizontal and equally rough vertical plane, the coefficient of friction being μ. Show that the greatest angle that the bounding diameter can make with the horizontal plane is

$$\sin^{-1}\frac{(3\pi}{4}\;\frac{\mu(1+\mu)).}{1+\mu^2}$$ 30

8. (a) Use Simplex method to solve the following LPP : 30

Maximize $Z = 60x + 50y$

Subject to

$x + 2y \leq 40$

$3x + 2y \leq 60$

$x, y \geq 0.$

(b) Write a program to calculate the series

$$S = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$ for $x = 0.1$. Use function subprogram to calculate the factorial. 30