FS – 20 / 15-16

Mathematics

Paper – II

Time : 3 hours

Full Marks : 200

The figures in the right-hand margin indicate marks.

*Candidates should attempt Q. No. 1 from Section – A and Q. No. 5 from Section – B which are compulsory and any **three** of the remaining questions, selecting at least **one** from each Section.*

SECTION – A

1. Attempt any **two** of the following : 20×2 = 40

   (a) Establish Lagrange's Interpolation formula.

   (b) Compute by Simpson's one-third rule the integral $\int_0^1 x^2(1-x)dx$ correct to three places of decimal, taking step length equal to 0.1. Also compute the absolute error and relative error.

(c) Solve :

$$(1 + 2x)^2 \frac{d^2y}{dx^2} - 6(1 + 2x)\frac{dy}{dx} + 16y = 8(1 + 2x)^2$$

2. (a) Compute the positive root of $x^3 - x - 0.1 = 0$ by Newton-Raphson Method, correct to six significant figures. 20

(b) Solve the system of equations by Gauss elimination method :

$$2x_1 + 3x_2 + x_3 = 9$$

$$x_1 + 2x_2 + 3x_3 = 6$$

$$3x_1 + x_2 + 2x_3 = 8$$

correct upto 3-significant figures. 20

3. (a) Compute y(0.2) from the equation $\frac{dy}{dx} = x - y$, $y(0) = 1$, taking $h = 0.1$, by Runge-Kutta method correct upto five decimal places 20

(b) Define Planar graph. Prove that Peterson graph is non-planar. Draw a 4-regular planar graph with minimum number of edges. 20

4. (a) Find a complete integral of $z^2(p^2z^2 + q^2) = 1$. 20

(b) Obtain the orthogonal trajectories of the family of circles $x^2 + y^2 = 2ax$ in polar form. 20

SECTION – B

5. Attempt any **two** of the following : $20 \times 2 = 40$

(a) Write an algorithm for computing ${}^nC_r$.

(b) Find the moment of inertia of a solid ellipsoid about any principal axis.

(c) Show that in any central orbit, the sectorial area traced out by the radius vector to the centre of force increases uniformly per unit of time.

6. (a) A uniform rod OA of length 2a, free to turn about its end O, revolves with uniform angular velocity ω about the vertical OZ through O and is inclined at a constant angle α to OZ. Find the value of α. 20

(b) Solve the following linear programming problem using the simplex method : 20

Maximize $Z = -2x_1 + x_2 + 3x_3$

Subject to $x_1 - 2x_2 + 3x_3 = 2$

$3x_1 + 2x_2 + 4x_3 = 1$

and $x_1, x_2, x_3 \geq 0$

7. (a) How the problem of degeneracy arises in a transportation problem ? Explain, how does one over-come it. 20

(b) Design a flowchart to arrange a set of n numbers in decending order. 20

8. (a) Find the potential function $\psi(x, y, z)$ in the region $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq c$ satisfying the conditions : 20

(i) $\psi = 0$ on $x = 0, x = a, y = 0, y = b, z = 0$

(ii) $\psi = f(x, y)$ on $z = c, 0 \leq x \leq a, 0 \leq y \leq b$

(b) Find the solution of the Laplace's equation $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = 0$. 20