

PHYSICS

Time allowed : Three Hours

Full Marks : 100

All questions carry equal marks.

Answer any FIVE questions.

1. (a) Using the Residue theorem, evaluate the integral,

$$\int_0^{\infty} \frac{x^{p-1}}{1+x} dx, \quad 0 < p < 1. \quad 6$$

- (b) Find all square roots of the matrix,

$$A = \begin{pmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{pmatrix}. \quad 6$$

- (c) Solve the following equation using Laplace transform :

$$t F''(t) + F'(t) + t F(t) = 0$$

where $F(0) = 1$ and $F'(0) = 0$. (The prime symbol indicates differentiation w.r.t. t). 8

2. (a) Solve the problem of linear harmonic oscillator using Hamilton Jacobi equation. 5
- (b) Find the total time derivative of $H = H(q, p, t)$ in terms of Poisson bracket, where the symbols have their usual meaning. 5
- (c) Give the relativistic generalization of Newton's second law and deduce the relation $E = mc^2$. 6+4

3. (a) State the postulate(s) of non-relativistic quantum mechanics concerning the results of measurement of an observable. 5
- (b) Find the position operator in the momentum representation and verify that it is Hermitian. 5
- (c) State and prove Wigner-Eckert theorem. 2+8
4. (a) What is negative temperature ? Find an expression for the specific heat at constant volume of a system which exhibit negative temperature and discuss its temperature dependence. 2+6+2
- (b) Starting from the grand canonical partition function, obtain an expression for the energy density in the frequency interval ν to $\nu + d\nu$ of black body radiation. Hence find the expressions of free energy and pressure of black body radiation. 6+4
5. (a) Derive an expression for the specific heat of a free electron gas and show that at low temperature the specific heat is proportional to temperature. 8
- (b) Derive the energy dispersion relation for lattice vibrations of a 1-dimensional chain of atoms. 6
- (c) Give an account of the Weiss theory. 6
6. (a) Discuss the vector model of LS coupling for non-equivalent valance electrons. 7
- (b) What do you mean by "Population inversion" ? Why is it necessary for laser radiation ? 6
- (c) Explain the physics of holography. 7

7. (a) Assuming the nucleus to be a degenerate Fermi gas of Z protons and N neutrons, find an expression for the total zero point kinetic energy of the nucleus. 10
- (b) Following Fermi's theory of beta decay derive an expression for the time rate of emission probability of an electron with momentum within a range p to $p+dp$. 10
8. (a) Consider a fixed bias circuit of an n-p-n transistor (silicon) in CE configuration where the collector and the base are connected to + 10 volt dc supply through two resistors of magnitude $5\text{ k}\Omega$ and $20\text{ k}\Omega$ respectively; the emitter is connected to the common point. The d.c. current gain of the transistor is 80. Find the values of I_C and I_B . Is it possible to use the circuit as voltage amplifier ? Explain your answer. 10
- (b) With a neat circuit diagram, explain the operation of a class-B push-full amplifier. 10
9. (a) Deduce the Newton-Raphson formula to find a root of the equation $f(x) = 0$ numerically. 10
- (b) Write a C program to integrate a given function $\phi(x)$ from a given value c to another given value d using Simpson's $1/3$ rule. 10

10. Write notes on any **TWO** of the following :—

- (a) BE condensation
- (b) Solid state microwave oscillators
- (c) Elementary particles
- (d) Relativistic electrodynamics.

2×10