JL – 37/14
Statistics
Paper – I

Time : 3 hours

Full Marks : 200

The figures in the right-hand margin indicate marks.

*Answer any **five** questions.*

1. (a) Define r^{th} moment and cumulants of a probability distribution. Derive the expression for the first 4 cumulants in terms of moments. 15

   (b) Define characteristic function. Derive the characteristic function of the normal distribution. 10

   (c) State the inversion theorem of characteristic function for a continuous random variable and also state uniqueness theorem. Derive the density function of a distribution whose characteristic function is given by $\frac{\lambda}{\lambda - it}$. 15

2. (a) Define multinomial distribution for the trinomial distribution. Derive the moment generating function, hence or otherwise derive the variance matrix for the trinomial distribution. 15

(b) Define the following :

(i) Convergence in probability

(ii) Convergence almost surely

(iii) Convergence in distribution

(iv) Convergence in r^{th} mean

Prove that convergence almost surely implies convergence in probability. Through an example demonstrate that the converse is not necessarily true. 10

(c) State and prove Khintchine's weak law of large member. Explain the use of this law. 15

3. (a) With examples distinguish between the following terms : 10

(i) Correlation

(ii) Regression

(b) Derive the distribution of the sample correlation coefficient r when $\rho = 0$, when the samples are taken from a Bivariate normal distribution. 20

(c) Given a time series data explain how to fit a polynomial curve to the data. 10

4. (a) Explain the following terms :

   (i) Partial Correlation Coefficient

   (ii) Multiple Correlation Coefficient

   (iii) Intraclass Correlation Coefficient

   (iv) Correlation Ratio

   Explain the utility of each of them. 10

(b) Given a random sample of size n from Normal distribution with mean μ and variance σ^2, derive the distribution of the sample variance. 20

(c) Given a random sample of size m and n from independent normal distributions with means μ_1 and μ_2 and common variance σ^2, describe the test for the following hypothesis :

   (i) $H_0 : \mu_1 \leq \mu_2$ Vs $H_1 : \mu_1 > \mu_2$

   (ii) $H_0 : \mu_1 = \mu_2$ Vs $H_1 : \mu_1 \neq \mu_2$

(iii) If m = n and the sample is a paired sample from a bivariate distribution, testing for $H_0 : \mu_1 = \mu_2$ Vs $H_1 : \mu_1 \neq \mu_2$. 10

5. (a) Derive Cramer Rao lower bound (C-R lower bound) for an unbiased estimator of $g(\theta)$. Given a random sample from Poisson distribution with mean λ, examine whether the following estimator is unbiased for $e^{-\lambda}$. And further check whether the variance of this estimator attains C-R lower bound

$$T(X_i, \ldots, X_n) = \frac{1}{n}\sum_{i=1}^{n} T(X_i), \text{ where}$$

$$T(X_i) = \begin{cases} 1 & \text{if} \quad X_i = 0 \\ 0 & \text{if} \quad X_i = 1, 2, \cdots\cdots \end{cases}$$

15

(b) Given a random sample of size n from Poisson (λ) distribution, derive uniformly minimum variance unbiased estimator (UMVUE) of $e^{-\lambda}$. Examine whether the variance of this estimator attains C-R lower bound. 15

(c) State and prove Rao-Blackwell-Lehmann-Scheffe theorem. 10

6. (a) Let U_g denotes the class of unbiased estimators of $g(\theta)$ and U_0 denotes the class of unbiased estimators of zero. Prove that an estimator $T(X) \in U_g$ is minimum variance unbiased estimator of $g(\theta)$ if and only if $Cov(T(X), U(X)) = 0$ where $U(X) \in U_0$. 10

(b) Let X be a random variable with probability mass function

$$f(x) = \begin{cases} \alpha & \text{if} \quad x = -1 \\ (1-\alpha)\alpha^x & \text{if} \quad x = 0, 1, 2, \cdots\cdots \end{cases}$$

Examine whether UMVUE of α exists. 15

(c) State Cramer-Rao-Huzurbazar theorem and prove the asymptotic normality of maximum likelihood estimator for a real parameter θ.

15

7. (a) State and prove Neyman-Pearson fundamental lemma. 15

(b) Given a random sample of size n from exponential distribution with density $f(x, \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, x > 0, \theta > 0$. Derive $100\alpha\%$ UMP test for testing $H_0 : \theta \leq \theta_0$ Vs $H_1 : \theta > \theta_0$. 10

(c) Given a random sample of size n from normal distribution with mean μ and variance σ^2, both are unknown, derive $100(1-\alpha)\%$ UMPU test for testing $H_0 : \mu = \mu_0$ Vs $H_1 : \mu \neq \mu_0$. 15

8. (a) Describe Mann-Whitney test stating clearly the null hypothesis, test statistic and decision rule. 15

(b) Describe median test. Clearly indicate when do you use median test and Mann-Whitney test. 15

(c) Defining run and run test, describe the uses of run test. 10