JL – 29/14

Physics

Paper – I

Time : 3 hours

Full Marks : 200

The figures in the right-hand margin indicate marks.

*Answer **all** questions.*

1. (a) Using the generating function for Hermite Polynomials

$$\phi(x, h) = e^{2xh - h^2} = \sum_{n=0}^{\infty} H_n(x) \frac{h^n}{n!},$$

show that $H_n'(x) = 2n H_{n-1}(x)$. 15

(b) Evaluate the following integral using Cauchy integral formula, where C is the circle $|z| = \frac{3}{2}$

$$\int_C \frac{4 - 3z}{z(z-1)(z-2)}.$$ 10

(c) Define representation of a group. What are the reducible and irreducible representations? 10

(d) Evaluate $\in_{23i} \in_{2i3}$. 5

OR

(e) Find the residue of the following : 10

(i) $\cot z$ at $z = 0$

(ii) $\frac{\sin z}{(1-z^4)}$ at $z = i$

(f) What is a group ? Define isomorphism and homomorphism. 10

(g) What are the transformational properties of (i) Contravariant vector and (ii) Covariant vector ? How does derivative of a covariant vector transform ? 10

(h) Find the singularities of Legendre equation $(1-x^2)y'' - 2xy' + \ell(\ell+1)y = 0$. 10

2. (a) What are action-angle variables ? Illustrate the Hamilton-Jacobi approach with example of one dimensional oscillator. 15

(b) What are canonical transformations ? Show

that the transformation given by $P = \frac{1}{2}(p^2 + q^2)$, $Q = \tan^{-1}\left(\frac{q}{p}\right)$ is canonical. 15

(c) Construct the Lagrangian and hence the equation of motion of a simple pendulum placed in a uniform gravitational field. 10

OR

(d) Determine the forced small oscillations of a system under a force $F(t) = at$ (a is a constant), assuming that at $t = 0$ the system is at rest and in equilibrium. 15

(e) What is the principle of least action ? Using it, derive the Lagrange's equation of motion. 15

(f) What is Coriolis effect ? What is its role in wind circulation ? 10

3. (a) State and explain four Maxwell equations. 15

(b) How does electromagnetic fields transform under Lorentz transformations ? What are the

two field invariants one can construct out of $\vec{E}$ and $\vec{B}$ fields ? 15

(c) Why day time clear sky appears blue ? Explain. 10

OR

(d) Describe the motion of a charged particle in (i) only electric field and (ii) only in the presence of magnetic field. 15

(e) Explain the statement that accelerated charges emit electromagnetic radiation. Discuss the case when relativistic charged particle is in circular motion. 15

(f) What are Kramers-Kronig dispersion relations ? What do they tell about propagation of electromagnetic waves in various media ? 10

4. (a) How does a quantum mechanical system evolve in the following : 15

(i) Schrodinger picture

(ii) Heisenberg picture

(iii) Interaction picture

(b) What are Clebsch-Gordon coefficients ? Obtain C. G. coefficients for $\vec{J}_1 = \frac{1}{2}$, $\vec{J}_2 = \frac{1}{2}$. 10

(c) Suppose that we have a quantum mechanical simple harmonic oscillator of mass m and spring constant k and that the oscillator is in its ground state with energy $E_0^{(0)} = \frac{1}{2}\hbar\omega$, $\omega = \sqrt{\frac{k}{m}}$. Let a spring whose spring constant b be added to along side the original one. By how much is the energy of the ground state increased ? 10

(d) Find $\int_{-3}^{3} \delta(x+7)x^2 \, dx$ and $\int_{-3}^{3} \delta(x+1)x^2 \, dx$. 5

OR

(e) Explain how invariance of a system under set of symmetries leads to conservation laws ? What are discrete symmetries and what are continuous symmetries ? 10

(f) Given two operators such that [A, B] = iC,

show that the uncertainties in A, B in any arbitrary state are related by $\Delta A \Delta B \geq \frac{1}{2} \langle C \rangle$. 10

(g) Write down the Hamiltonian energy eigenvalue spectrum for non-relativistic hydrogen atom. Draw its ground state wave function. 10

(h) What happens to the energy spectrum of an hydrogen atom, if it is placed in (i) magnetic field only and (ii) electric field only ? Explain. 10

5. (a) What is Gibb's paradox ? Discuss the method to resolve this paradox. 5+10 = 15

(b) Derive the equation of state for an ideal Fermi gas and the conditions leading to the complete degeneracy of this system. 15

(c) What are phase transitions ? Discuss with an example in each case. 10

OR

(d) What do you understand by the term ensemble ? Discuss the importance of ensemble concept and various types of ensembles. 15

(e) Explain the following terms : 15

(i) Bose-Einstein distribution

(ii) Fermi-Dirac distribution

(iii) Bose-Einstein condensation

(f) Define phase space of a classical system. Describe how the microstate of a physical system is represented in such a space. 10