JL – 25/14
Mathematics
Paper – I

Time : 3 hours

Full Marks : 200

The figures in the right-hand margin indicate marks.

*Answer **all** questions.*

1. (a) In a Group G, if a is an element of order n and p is prime to n then show that a^p is also of order n.

OR

If G is a finite group of order N and $a \in G$ then show that $a^n = e$. State the theorem you will use here. Here e is the identity element.

(b) A ring R is without zero divisors if cancellation law holds in R.

OR

If R is a ring such that $a^2 = a$ for all $a \in R$ then show that R is a commutative ring.

(c) Prove that every integral domain is a field.

OR

Show that the polynomial $(8x^3 - 6x - 1)$ is irreducible over the field of rational numbers.

(d) If two integers a and b are relatively prime i.e. if $(a, b) = 1$ then show that $a|bc \Rightarrow a|c$.

OR

State and prove Lagrange's Theorem for number theory. 40

2. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at origin though the Cauchy-Riemann equations are satisfied at the point, where $z = x + iy$.

OR

Show that an analytic function with constant modulus is constant.

(b) Obtain the bilinear transformation which maps the points $z_1 = 2$, $z_2 = i$, $z_3 = -2$ to $w_1 = 1$, $w_2 = i$, $w_3 = -1$ respectively.

OR

Find a transformation which maps outside $|z| = 1$, on the half plane $Re(w) \geq 0$, so that the points $z = 1, -i, -1$ correspond to $w = i, 0, -i$ respectively.

(c) Evaluate $\int_c \frac{e^{az}}{z^2 + 1} dz$ where c is the circle $|z| = 2$.

OR

Show that $\int_0^\infty \frac{\cos mx}{a^2 + x^2} dx = \frac{\pi}{2} e^{-ma}$, $m \geq 0$ using Contour integration.

(d) What kind of singularity following functions have $\frac{\cot \pi z}{(z - a)^2}$ at $z = 0$ and $z = a$ and find the residue at $z = a$.

OR

Evaluate $\int_c \frac{z - 3}{z^2 + 2z + 5} dz$, where c is the circle $|z + 1 + i| = 2$. 40

3. (a) Prove that in a metric space, every closed sphere is a closed set.

OR

If $\overline{T}$ is the collection of subsets of N containing empty set ϕ and all subsets in the form

$G_m = \{m, m + 1, m + 2 \text{-------}\}$ $m \in N$ show that $\overline{T}$ is topology for N.

(b) Show that the sequence $\{S_n\}$ where

$$S_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots\cdots\cdots + \frac{1}{n+n},$$

for all $n \in N$ is convergent.

OR

Prove that every convergent sequence is bounded.

(c) Show that the continuous image of a BWP set is BWP.

OR

Let f be a continuous mapping of a compact metric space (X, d) into a metric space (Y, p) then f is uniformly continuous on X.

(d) Evaluate $\iint (y - x)\, dx\, dy$ over the region R where R is bounded by the lines $y = x - 3$, $y = x + 1$, $3y + x = 5$, $3y + x = 7$.

OR

Let $f(x, y) = \dfrac{xy\left(x^2 - y^2\right)}{x^2 + y^2}$, $(x, y) \neq (0, 0)$

$= 0$, $(x, y) = 0$

Show that at the origin $f_{xy} \neq f_{yx}$. 40

4. (a) Show that the following system of linear equations has a degenerate solution :

$$2x_1 + x_2 - x_3 = 2$$
$$3x_1 + 2x_2 + x_3 = 3$$

OR

Let $x_1 = 2$, $x_2 = 4$ and $x_3 = 1$ be a feasible solution to the system of equations :

$$2x_1 - x_2 + 2x_3 = 2$$
$$x_1 + 4x_2 \qquad = 18$$

Reduce the given feasible solution to a basic feasible solution.

(b) Consider the L. P. P. :

Maximize $Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

Subject to the constraints

$$x_1 + x_2 + x_3 = 4$$

$$x_1 + 4x_2 + x_4 = 8$$

$$x_1, x_2, x_3, x_4 \geq 0$$

By using x_3 and x_4 as the starting variables, the optimum table is given by :

Basics	**Solution**	x_1	x_2	x_3	x_4
x_3	2	$\frac{3}{4}$	0	1	$-\frac{1}{4}$
x_2	2	$\frac{1}{4}$	1	0	$\frac{1}{4}$
z	16	2	0	0	3

Write the dual problem and find its solution from the optimum primal table.

OR

State and prove Complementary Slackness Theorem.

(c) Determine the range of value of p and q that will make the payoff element a_{22}, a saddle point for the game whose payoff matrix (a_{ij}) is given below :

$$\text{Player A} \overset{\text{Player B}}{\begin{bmatrix} 2 & 4 & 7 \\ 10 & 7 & q \\ 4 & p & 8 \end{bmatrix}}$$

OR

For the game with the following payoff matrix, determine the optimum strategies and the value of the game.

$$P_1 \overset{P_2}{\begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix}}$$

(d) Obtain an initial basic feasible solution to the following transportation problem using the north-west corner rule :

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

OR

A student has to select one and only one elective in each semester and the same elective should not be selected in different semesters. Due to various reasons, the expected grades in each subject, if selected in different semesters, vary and they are given below :

Semester	Analysis	Statistics	Graph Theory	Algebra
I	F	E	D	C
II	E	E	C	C
III	C	D	C	A
IV	B	A	H	H

The grade points are : H = 10, A = 9, B = 8, C = 7, D = 6, E = 5 and F = 4. How will the student select the electives in order to maximize the total expected points and what will be his maximum expected total points ? 40

5. (a) Find the roots of the equations $x^2 - \cos x = 0$ by Newton-Raphson's method. Correct upto three places of decimal.

OR

Using Newton-Raphson method find the solution of the following system of equations near $x = 2, y = 1$. Correct upto two places of decimal only :

$$x^2 + y^2 - y = 5$$

$$y - e^{-x} = 1$$

(b) Following table gives the value of f(x) for $x = 2.00\ (0.05)\ 2.25$:

x	f(x)
2.00	0.69315
2.05	0.71784
2.10	0.74194
2.15	0.76547
2.20	0.78846
2.25	0.81093

Compute the value of f(2.07) using appropriate formulae.

OR

Using Lagrange's formula, find the value of y(2) from the following data :

x	y
0	8
1	11
4	68
5	123

(c) Evaluate the integral $I = \int_0^{1.2} e^{-x^2} dx$ by Simpson's Rule. Take h = 0.2. Also estimate the error in the Simpson's Rule.

OR

Find the minimum number of intervals that will be required to evaluate the integral

$I = \int_0^1 \frac{dx}{1+x}$ by Simpson's Rule, so that error

does not exceed by 10^{-4}. Evaluate the integral.

(d) Compute y at x = 0.2 (0.2) 0.4 by fourth order Runge-Kutta method for the differential equation.

$$\frac{dy}{dx} = (y - x, y ; 0) = 1.5$$

Give your answer upto four places of decimal.

OR

Solve the differential equation $\frac{dy}{dx} = x^2 + y^2 - 2$, for x = 0.3, by Milne's predictor-corrector method. Compute the starting values at x = − 0.1, 0, 0.1 and 0.2 by Taylor's expansion about x = 0, where y(0) = 1 taking first four non-zero terms. Show your calculations upto four decimals only. 40