1. If $f : \mathbb{R}^2 \to \mathbb{R}$ is a linear transformation, then which of the following does not hold?
   A) f(1, 1) = 2 and f (2,1) = 3 B) f(1, 1) = 3 and f (2,2) = 3
   C) f(1, 1) = 4 and f (2,3) = 5 D) f(1, 1) = 5 and f (3,2) = 5

2. Let $V = \mathbb{R}^3$ be a vector space over $\mathbb{R}$. Which of the following is a subspace of V?
   A) $W = \{(1,2,x) : x \in \mathbb{R}\}$ B) $W = \{(0,1,x) : x \in \mathbb{R}\}$
   C) $W = \{(1,0,x) : x \in \mathbb{R}\}$ D) $W = \{(x,0,x) : x \in \mathbb{R}\}$

3. Let $f:\mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation given by f(x,y,z)=(x-y, x-y, y+z).Rank f =
   A) 0 B) 1 C) 2 D) 3

4. Find the dimension of the space of solutions of the following system of equations
   $x + y + z = 0$
   $2x + 3y + z = 0$
   $3x + 2y +4z = 0$
   A) 1 B) 2 C) 3 D) 4

5. Which of the following is not a characteristic value of the linear transformation $f : \mathbb{R}^3 \to \mathbb{R}^3$ given by f(x,y,z) = (x+y, x+z, y+z)?
   A) 0 B) -1 C) 1 D) 2

6. Let $A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
   Then the minimal polynomial of A is
   A) (x-1) (x-2) B) $(x-1)(x-2)^2$
   C) $(x-1)^2(x-2)$ D) $(x-1)^2(x-2)^2$

7. Which of the following polynomials is a minimal polynomial of a 4x4 diagonalizable matrix?
   A) $(x-1)^4$ B) $(x-1)^2(x-2)^2$
   C) $(x-1)(x-2)^2$ D) (x-1) (x-2) (x - 3)

8. If α_1, α_2 are orthogonal vectors in an inner product space; then which of the following is true?
   A) $\|\alpha_1 + \alpha_2\| = \|\alpha_1\| + \|\alpha_2\|$ B) $\|\alpha_1 + \alpha_2\|^2 = \|\alpha_1\|^2 + \|\alpha_2\|^2$
   C) $\|\alpha_1 - \alpha_2\| = \|\alpha_1\| - \|\alpha_2\|$ D) $\|\alpha_1 - \alpha_2\|^2 = \|\alpha_1\|^2 - \|\alpha_2\|^2$

9. Let $\mathbb{R}^2$ be the inner product space with usual inner product. Which of the following is in the orthogonal complement of $W = \{(x,x) : x \in \mathbb{R}\}$?
   A) (1,0) B) (0,1) C) (1,-1) D) (1,2)

10. Which of the following is a self adjoint operator on $\mathbb{R}^2$?

A) f (x,y) = (x+y, 2x+y) B) f (x,y) = (x+2y, x+y)

C) f (x,y) = (x+2y, 2x+y) D) f (x,y) = (x+2y, x+2y)

11. Let G = {$\pm$ 1, $\pm$ i, $\pm$ j, $\pm$ k} be the group of quaternions. If φ:G $\rightarrow$ G is a homomorphism with φ (i) = j and φ (j) = i then φ (k) =

A) 1 B) -1 C) k D) -k

12. Let φ be an automorphism of the cyclic group Z_{12}. Then which of the following can not be a value of φ (1)?

A) 2 B) 5 C) 7 D) 11

13. Let S_3 be the symmetric group on three symbols. Then the number of conjugacy classes of S_3 is

A) 1 B) 2 C) 3 D) 4

14. The number of 3-sylow subgroups of a group of order 15 is

A) 1 B) 3 C) 4 D) 5

15. Which of the following is not a maximal ideal in the ring Z_{18} of integers mod 18?

A) The ideal generated by 4 B) The ideal generated by 6

C) The ideal generated by 8 D) The ideal generated by 10

16. Let m, n be positive integers and g.c.d of m and n be d = λm +   n, for some integers λ,   , where d >1. Then which of the following is true?

A) g.c.d of λ and   is d B) g.c.d of λ and   is 2d

C) λ and   are relatively prime D) λ and   are positive

17. Let Z [i] be the ring of Gaussian integers. If a + bi is a unit in Z [i] then

A) $a^2 + b^2 = 1$ B) $a^2 + b^2 > 1$

C) $a^2 + b^2 < 1$ D) $a^2 = b^2$

18. Let R be a Euclidean ring with Euclidean valuation ε. Let u be a unit in R and a $\neq$ 0 be a non unit in R. Then which of the following is necessarily false?

A) ε (u) $\leq$ ε (a) B) ε (ua) = ε (u)

C) ε (u) = ε (u^2) D) ε (u) = ε (1)

19. The degree of the field extension $Q\left(\sqrt{2}+\sqrt{3}\right)$ over Q is

A) 1 B) 2

C) 3 D) 4

20. Which of the following pairs of fields are isomorphic?

A) Q ($\sqrt{2}$) and Q ($\sqrt{3}$)

B) Q ($\sqrt{2}$ + $\sqrt{3}$) and Q ($\sqrt{6}$)

C) Q ($\sqrt{2}$, $\sqrt{3}$) and Q ($\sqrt{2}+\sqrt{3}$)

D) Q ($\sqrt{2}$, $\sqrt{3}$) and Q ($\sqrt{5}$)

21. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^2 + 4x + 2$ and X be a compact subset of $\mathbb{R}$. Then f (X) is ----------------

A) A closed and bounded subset of $\mathbb{R}$

B) A bounded but not closed subset of $\mathbb{R}$

C) A closed but not bounded subset of $\mathbb{R}$

D) Neither a closed nor a bounded subset of $\mathbb{R}$

22. The Fourier series expansion for the function $f(x) = x^2$ in the interval $[-\Pi, \Pi]$ is ----

A) $\frac{\Pi}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n x$

B) $\frac{\Pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n x$

C) $\frac{\Pi}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \sin n x$

D) $\frac{\Pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \sin n x$

23. The entremum value of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$ is ----------

A) -8

B) 8

C) -7

D) 7

24. The value of the double integral $\int_0^3 \int_0^2 (4 - y^2)\, dy\, dx$ is -------------

A) 16

B) 6

C) $\frac{32}{3}$

D) None of the above

25. Which of the following subset of $\mathbb{R}$ is a complete metric space with respect to the usual metric on $\mathbb{R}$?

A) Set of rationals

B) Set of irrationals

C) [1, 2]

D) $(0, \infty)$

26. Which of the following sequence of functions is uniformly convergent on the interval $[0, \infty)$?

A) $f_n(x) = \frac{nx}{1+n^2x^2}$

B) $f_n(x) = \frac{nx}{1+nx}$

C) $f_n(x) = \frac{\sin nx}{1+nx}$

D) $f_n(x) = x^2 e^{-nx}$

27. The set of irrational numbers in the interval (3, 5] is of measure ---------------

A) 0

B) 1

C) 2

D) None of the above

28. Let f : (4, 8) → R be defined by f (x) = x + 5. Which of the following is a true statement?
A) f is continuous and measurable
B) f is continuous but not measurable
C) f is measurable but not continuous
D) f is neither continuous nor measurable

29. Let f be a non-negative measurable function defined on [5, 10] and let $\int_5^{10} f\,dx = \alpha$

If $$g(x) = \begin{cases} f(5)+5 & \text{if } x=5 \\ f(7)+7 & \text{if } x=7 \\ f(9)+9 & \text{if } x=9 \\ f(x) & \text{otherwise,} \end{cases}$$

Then $\int_5^{10} g\,dx$ = ------------------

A) $\alpha + 5$ B) $\alpha + 7$
C) $\alpha + 9$ D) α

30. If $f_n(x) = \frac{nx}{1+n^2x^2}$, then $\lim_{n \to \infty} \int_0^1 f_n(x)\,dx$ is --------------
A) 0 B) 1
C) ½ D) None of the above

31. Let $Y = \{(k_1, k_2) \in \mathbb{R}^2 : k_1 - 3k_2 = 0\}$ be a normed space with norm $\|(k_1, k_2)\|_1 = |k_1| + |k_2|$ and $g : Y \to \mathbb{R}$ be defined by $g(k_1, k_2) = k_1$. Which of the following f is a Hahn Banach extension of g to $\mathbb{R}^2$?
A) $f(k_1, k_2) = \frac{1}{4}(k_1 + k_2)$ B) $f(k_1, k_2) = \frac{1}{4}k_1 + \frac{3}{4}k_2$
C) $f(k_1, k_2) = \frac{3}{4}(k_1 + k_2)$ D) $f(k_1, k_2) = \frac{3}{4}k_1 + \frac{1}{4}k_2$

32. Let X denote the Banach space $\mathbb{R}^3$ with respect to the norm $\|(x(1), x(2), x(3))\|_1 = |x(1)| + |x(2)| + |x(3)|$ and let $A\big((x(1), x(2), x(3))\big) = \big(2x(1), 3x(2), 4x(3)\big)$ be an operator on X. Then $\|A\|_1$ = ------------------
A) 2 B) 3 C) 4 D) 9

33. Let X, Y be normed spaces, X ≠ {0} and BL (X,Y) be the set of all bounded linear maps from X to Y. Then BL (X,Y) is a Banach space under the operator norm if ---
A) X is a Banach space B) Y is a Banach space
C) Y ≠ {0} D) None of the above

34. Let X be a finite dimensional Banach space over R with norm $\| \ \|$. Let $F : X \to X$ be defined by F (x) = 2x. Then F is ------------------

A) Continuous and open B) Continuous but not open
C) Open but not continuous D) Neither open nor continuous

35. Let E be an orthonormal set in a Hilbert space H. Then E is ---------------

A) Linearly independent and $\|x-y\| = \sqrt{2}$ for all $x, y \in E$
B) Linearly dependent and $\|x-y\| = \sqrt{2}$ for all $x, y \in E$
C) Linearly independent and $\|x-y\| = 2$ for all $x, y \in E$
D) Linearly dependent and $\|x-y\| = 2$ for all $x, y \in E$

36. Let $X = \mathbb{R}^2$ be the inner product space over $\mathbb{R}$ with inner product $\langle (x(1), y(1)), (x(2), y(2)) \rangle = x(1)\ x(2) + y(1)\ y(2)$ and let $Y = \{(x, y) : x = y\}$ be a subspace of X. Which of the following is the orthogonal complement of Y ($Y^{\perp}$)?

A) Y B) $\{(x, -x) : x \in \mathbb{R}\}$
C) $\{(x, 0) : x \in \mathbb{R}\}$ D) $\{(0, x) : x \in \mathbb{R}\}$

37. Let C denote the set of complex numbers and C^3 the complex Hilbert space with respect to the inner product

$$< x, y > = \sum_{j=1}^{3} x(j)\ \overline{y(j)}\ , \quad x, y \in C^3$$

Then the representor of the linear functional $f : C^3 \to C$ defined by $f(z_1, z_2, z_3) = z_1 + iz_2 - iz_3$ is ------------------

A) (1, i, i) B) (1, i, -i) C) (1, -i, i) D) (1, -i,-i)

38. Let C denote the set of complex numbers and H be the Hilbert space C^2 with respect to the usual inner product (given in the above problem). If $A : H \to H$ be defined by A (x(1), x(2)) = (2x(1), x(2)), then A^* is --------------

A) A B) -A C) iA D) -iA

39. Let H be Hilbert space. Which of the following is not a correct statement?

A) Every unitary operator on H is normal
B) The set of normal operators is a closed subset of the set of bounded operators on H
C) A is normal if and only if $\|A(x)\| = \|A^*(x)\|$ for all $x \in H$
D) The eigen vectors corresponding to distinct eigen values of a normal operator are not orthogonal

40. Let H be a Hilbert space with inner product <, > and y be a fixed element in H. Then the linear functional f on H defined by f(x) = <x, y> has norm -------------

A) $\|x\|$ B) $\|y\|$
C) $\|x + y\|$ D) None of the above

41. For which of the following values of the complex number α, does the Mobius transformation f_α defined by $f_\alpha(z) = \dfrac{z-\alpha}{1-\bar{\alpha} z}$ map the unit disk $D = \{z : |z| < 1\}$ onto itself

A) 1 B) ½ C) i D) -i

42. Let r be the path defined by $r(t) = 2e^{it}$, $0 \le t \le 2\pi$. Then the value of the integral $\int_r \dfrac{dz}{z^2+1}$ is

A) 0 B) $4\pi i$ C) $2\pi i$ D) $-2\pi i$

43. At z = 0, $\dfrac{\sin z}{z^3} - \dfrac{1}{z^2}$ has a

A) Pole of order two B) Simple pole
C) Removable singularity D) Essential singularity

44. The value of the real integral $\int_0^\infty \dfrac{\sin x}{x}\,dx$ is

A) 0 B) π C) 2π D) $\dfrac{\pi}{2}$

45. For $z \in \mathbb{C}$, $z \neq 0$, let $f(z) = e^{1/z} - \dfrac{2}{z}$. Then the residue of f at z =0 is

A) 1 B) 0 C) -1 D) -2

46. The Taylor series expansion of the function $f(z) = \dfrac{1}{2z-1}$ about z = 0 has radius of convergence

A) ½ B) 1 C) 2 D) -1

47. The coefficient of $\dfrac{1}{z}$ in the Laurent series expansion of $f(z) = \dfrac{1}{(z-1)(z-2)}$ in the annulus $1 < |z| < 2$ is

A) 0 B) 1 C) -1 D) 2

48. Which of the following subsets of the complex plane is not simply connected?

A) $\{z : \text{Re } z > 0\}$
B) $\{z : |z| < 1\}$
C) $\{z : |z-1| < 2\} \cup \{z : |z+1| < 2\}$
D) $\{z : |z| > 1\}$

49. $\int_r \left(\dfrac{z}{z-1}\right)^3 dz$, where $r(t) = 1 + e^{it}$, $0 \le t \le 2\pi$ has the value

A) 0 B) $2\pi i$

C) $4\pi i$ D) $6\pi i$

50. If f is analytic on $D = \{z : |z| < 1\}$ and if $|f(z)| \le 1$ for all z in D and $f(0) = 0$, then it is always true that

A) $|f(z)| \le ½$, when $|z| \le ½$ B) $|f^1(0)| \le ½$

C) $|f(z)| \le ½ |z|$ for $|z| \le ½$ D) $f^1(0) = 0$

51. The value of the integral

$\int_0^1 \dfrac{dx}{1+x}$ if evaluated using Simpson's rule with step size h = ½ is

A) $\frac{2}{3}$ B) $\frac{25}{36}$ C) $\frac{13}{18}$ D) $\frac{3}{4}$

52. The eigen values of the matrix $A = \begin{bmatrix} 1/2 & 1 \\ 1 & -1/3 \end{bmatrix}$ are

A) 1 and $\frac{7}{6}$ B) -1 and $\frac{7}{6}$ C) -1 and $\frac{-7}{6}$ D) 1 and $\frac{-7}{6}$

53. The unique polynominal of degree ≤ 2 which interpolates to the polynomial x^5 at the points 0, 1 and -1 is

A) $x^2 - x$ B) $x^2 + x$ C) x D) x^2

54. Let p(x) be the unique linear interpolant to the function e^x interpolated at the points $\pm$ 1. Then for some c, $-1 < c < 1$, the error $e^x - p(x)$ is given by

A) $\dfrac{x^2 - 1}{2} e^c$ B) $\dfrac{1 - x^2}{2} e^c$ C) $\dfrac{x^2 - 1}{3} e^c$ D) $\dfrac{x^2 + 1}{2} e^c$

55. For the events A and B, let P(A) = p, P(A/B) = q, P(B/A) = r. If $\tilde{A}$ and $\tilde{B}$ are mutually exclusive, then p, q, r satisfy the relation

A) pq + r = 1 B) p(q+r) = q(1+pr)

C) p(q-r) = q D) (q + r) p = qr

56. Three coins are tossed simultaneously. Assuming the probability of getting a head and the probability of not getting a head are equal, what is the probability of getting at least one head?

A) ⅛ B) ¼ C) ⅞ D) ½

57. The mode of the Poisson distribution $p(x, \lambda) = \begin{cases} \frac{e^{-\lambda} \lambda^x}{x!}, & x = 0,1,2,---; \lambda > 0 \\ 0, & \text{otherwise} \end{cases}$ is

A) $\frac{x}{\lambda}$ B) $\frac{\lambda}{x}$ C) $\frac{x^2}{\lambda}$ D) $x\lambda$

58. The probability distribution of a random variable X is : $f(x) = k \sin \frac{\pi x}{5}, 0 \leq x \leq 5$.

Then the value of the constant k is

A) $\frac{\pi}{5}$ B) $\frac{\pi}{3}$ C) $\frac{\pi}{10}$ D) $\frac{\pi}{2}$

59. The closed sets in an infinite set X with the cofinite topology are

A) ϕ and X only
B) ϕ, X and all finite sets in X
C) ϕ, X and all infinite sets in X
D) All subsets of X

60. The β^{th} projection map $\Pi_\beta : \Pi X_\alpha \to X_\beta$ is

A) Continuous, open and closed
B) Continuous and open but need not be closed
C) Continuous but need not be open and closed
D) Need not be continuous, open and closed

61. If A^0 denotes the interior of A in a topological space X and if A and B are subsets of X, then

A) $A^0 = A$ B) $(A^0)^0 = \phi$
C) $(A \cup B)^0 = A^0 \cup B^0$ D) $(A \cap B)^0 = A^0 \cap B^0$

62. A topological space having a countable open base at each of its points is said to be

A) First countable B) Second countable
C) Lindelöf D) Regular

63. Consider the following statements

I. Any subspace of a discrete space is discrete
II. Any subspace of a normal space is normal.

Then

A) Both statements I and II are true
B) Statement I is true and statement II is false
C) Statement I is false and statement II is true
D) Both statements I and II are false

64. The real line R with the usual topology is
A) Compact and locally compact
B) Compact but not locally compact
C) Not compact but locally compact
D) Neither compact nor locally compact

65. The components of a topological space X are
A) Both open and closed
B) Open but need not be closed
C) Closed but need not be open
D) Neither open nor closed

66. Consider the following statements:
I. Every continuous map from a compact space to a Hausdorff space is a closed map.
II. A compact Hausdorff space is a T_4-space.
Then
A) Both statements I and II are true
B) Statement I is true and statement II is false
C) Statement I is false and statement II is true
D) Both statements I and II are false

67. Which of the following statements is always true?
A) Every pathwise connected space is connected
B) Every connected space is pathwise connected
C) Every locally connected space is connected
D) Every connected space is locally connected

68. Let $f : X \rightarrow Y$ be a continuous mapping of a topological space X to another topological space Y. Choose the statement which is not correct?
A) $f^{-1}(H)$ is open in X whenever H is open in Y
B) $f^{-1}(K)$ is closed in X whenever K is closed in Y
C) For each subset E of X, $f(Cl_x E) \subset Cl_Y f(E)$
D) For each subset E of X, $f(Cl_x E) = Cl_Y f(E)$

69. The number of basic solutions to the system
$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$
$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$ is

A) 2
B) 4
C) 6
D) 8

70. A basic solution to a system of linear equations in a linear programming problem in which one or more of the basic variables become equal to zero is called a
A) Feasible solution
B) Optimal solution
C) Basic feasible solution
D) Degenerate solution

71. A firm makes two products : chairs and tables. Processing of these products is done on two machines X and Y. A chair requires 2 hours on machine X and 6 hours on machine Y. A table requires 5 hours on machine X and 3 hours on machine Y. There are 16 hours of time per day available on machine X and 30 hours on machine Y. Profit gained from a chair and a table is Rs.7 and Rs.10 respectively. If x_1 and x_2 denotes the number of chairs and tables respectively, the objective function of the linear programming problem formulated is

A) Maximize $7x_1 + 10x_2$ B) Maximize $8x_1 + 8x_2$
C) Maximize $2x_1 + 5x_2$ D) Maximize $2x_1 + 6x_2$

72. The linear programming problem

maximize $Z = 3x_1 + 2x_2$
subject to $x_1 - x_2 \leq 1$
$x_1 + x_2 \geq 3$
$x_1, x_2 \geq 0$ has

A) A unique solution B) An infinite number of solutions
C) An unbounded solution D) No feasible solution

73. The optimal value of the objective function of the problem

minimize $W = x_1 + 4x_2 + 2x_3 + 8x_4$
subject to $x_1 + x_2 + x_3 + x_4 \leq 6$
$x_i \geq 0$; $i = 1, 2, 3, 4$ is

A) 6 B) 3 C) 2 D) 0

74. Consider the transportation problem

From \ To	D1	D2	D3	Supply
S1	6	8	4	14
S2	4	9	8	12
S3	1	2	6	5
Demand	6	10	15	

The transportation cost when initial basic feasible solution is obtained by least cost method is

A) 160 B) 164
C) 163 D) None of these

75. The minimum cost for the transportation problem

	D1	D2	D3	Available
O_1	30	20	10	800
O_2	5	15	25	500
Required	300	300	400	

is

A) 10000 B) 10500
C) 11000 D) None of these

76. A department head has 4 subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. The estimate of time each man would take to perform each task is given below

Jobs \ Men	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

Assume each man is assigned only one job and each job is assigned only one man. The minimum time taken to complete the 4 jobs by 4 men is

A) 64 B) 52 C) 41 D) 30

77. The objective function of the dual of the following primal problem

minimize $Z = 5x_1 + 8x_2$

subject to $4x_1 + 9x_2 \geq 100$

$2x_1 + x_2 \leq 20$

$2x_1 + 5x_2 \geq 120$

$x_1, x_2 \geq 0$ is

A) Maximize $Z^1 = 100y_1 + 20y_2 + 120y_3$

B) Maximize $Z^1 = 100y_1 - 20y_2 + 120y_3$

C) Maximize $Z^1 = 4y_1 + 2y_2 + 2y_3$

D) Maximize $Z^1 = 4y_1 - 2y_2 + 2y_3$

78. If $x_1 = 0$, $x_2 = 0$, $x_3 = 4$ and $x_4 = 4$ is a feasible solution to the linear programming problem (primal)

maximize $Z = x_1 + 2x_2 + 3x_3 + 4x_4$

subject to $x_1 + 2x_2 + 2x_3 + 3x_4 \leq 20$

$2x_1 + x_2 + 3x_3 + 2x_4 \leq 20$

$x_1, x_2, x_3, x_4 \geq 0$

and if $y_1 = 1.2$ and $y_2 = 0.2$ is a feasible solution to its dual given by

minimize $W = 20y_1 + 20y_2$

subject to $y_1 + 2y_2 \geq 1$

$2y_1 + y_2 \geq 2$

$2y_1 + 3y_2 \geq 3$

$3y_1 + 2y_2 \geq 4$

$y_1, y_2 \geq 0$,

Then consider the statements

I. The given feasible solution to the primal is an optimal solution to the primal.

II. The given feasible solution to the dual is an optimal solution to the dual.

Of these

A) Both statements I and II are true

B) Statements I is true and statement II is false

C) Statement I is false and statement II is true

D) Both statements I and II are false

79. The general solution of the equation y sin (xy) dx + x sin (xy) dy = 0 is

A) xy sin xy = c B) xy cos xy = c
C) (x+y) sin xy = c D) cos xy = c

80. The general solution of the equation $xdy - ydx = 2(x^2+y^2)dx$ is

A) $\tan^{-1}(y/x) = 2x+c$ B) $\tan^{-1}(x/y) = 2x+c$
C) $\tan^{-1}(xy) = 2x+c$ D) $\tan^{-1}(x+y) = 2x+c$

81. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solution of the equation $y^{11} + P(x)y^1 + Q(x)y = R(x)$. If $W(y_1,y_2)$ is the Wronskian of $y_1(x)$ and $y_2(x)$ then a solution of the above differential equation is

A) $y_1\int \frac{-y_2 R(x)}{w(y_1,y_2)}\,dx + y_2\int \frac{y_1 R(x)}{w(y_1,y_2)}\,dx$

B) $y_1\int \frac{y_2 R(x)}{w(y_1,y_2)}\,dx + y_2\int \frac{y_1 R(x)}{w(y_1,y_2)}\,dx$

C) $y_1\int \frac{y_1 R(x)}{w(y_1,y_2)}\,dx + y_2\int \frac{y_2 R(x)}{w(y_1,y_2)}\,dx$

D) $y_1\int \frac{y_1 R(x)}{w(y_1,y_2)}\,dx + y_2\int \frac{-y_2 R(x)}{w(y_1,y_2)}\,dx$

82. If $D = \frac{d}{dx}$, $D^2 = \frac{d^2}{dx^2}$ etc, then the solution of the equation $[(D^2 - 2D + 1)(D^4 + 2D^2 + 1)]\,y = 0$ is

A) $y = (c_1 + c_2x)\,e^x + (c_3 + c_4x)(\cos x + \sin x)$
B) $y = (c_1 + c_2x)\,e^x + c_3 \cos x + c_4 \sin x$
C) $y = (c_1 + c_2x)\,e^x + (c_3 + c_4x)\cos x + (c_5 + c_6x)\sin x$
D) $y = (c_1 + c_2x)\,e^x\,(c_3 \cos x + c_4 \sin x)$

83. If $P_n(x)$ denotes the n^{th} degree Legendre polynomial then $P_6(0)$ is

A) $\frac{5}{16}$ B) $\frac{-5}{16}$
C) $\frac{15}{16}$ D) $\frac{-15}{16}$

84. $\int\left[x^4 J_3(x) + x^{-4} J_5(x)\right]dx$ is

A) $2x^4 J_4(x) + c$ B) $2x^{-4} J_4(x) + c$
C) $(x^4 - 1/x^4)\,J_4(x) + c$ D) $(x^4 + 1/x^4)\,J_4(x) + c$

85. The number of irregular singular points for the equation

$x^2(x^2-1)^2\frac{d^2y}{dx^2} - x(1-x)\frac{dy}{dx} + 2y = 0$ is

A) 1 B) 2 C) 3 D) 0

86. The number of regular singular points of the hyper geometric equation $x(1-x)y^{11} + [c-(a+b+1)x]y^{1} - aby = 0$ is

A) 0 B) 1 C) 2 D) 3

87. The Wronskian of the two solution ($x = e^{3t}$, $y = e^{3t}$) and ($x = e^{2t}$, $y = 2e^{2t}$) of the linear system $\frac{dx}{dt} = 4x - y\,;\ \frac{dy}{dt} = 2x + y$ is

A) $3e^{5t}$ B) e^{5t}
C) $6e^{5t}$ D) $e^{6t} - e^{4t}$

88. The equation $x^2 U_{xx} + 2x U_{xy} + U_{yy} = U_y$ is

A) Elliptic B) Hyperbolic
C) Parabolic D) Cyclic

89. If $4\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0;\ -\infty < x < \infty,\ t > 0;$ is the one dimensional wave equation, then the domain of dependence of the point (1,2) is the line segment

A) from (0,0) to (1,2) B) from (1,0) to (2,0)
C) from (0,1) to (0,2) D) from (-3,0) to (5,0)

90. Let D be the interior of a simple, closed and piecewise smooth curve B. Let u_1 and u_2 be two solutions of the problem : $\nabla^2 u = 0$ in D; $\frac{\partial u}{\partial n} = f(s)$ on B, where $\frac{\partial}{\partial n}$ is the directional derivative along the outward normal. Then

A) $u_1 = u_2$ B) $u_1 = cu_2$, where c is a constant
C) $u_1 + u_2 = 0$ D) $u_1 = u_2 + c$ where c is a constant

91. Let u (x,y) be the solution of the problem:

$$u_{xx} + u_{yy} = 0,\ 0 < x < a,\ 0 < y < b$$

with the boundary conditions

$$u(x, 0) = f(x)\ ,\ 0 \le x \le a$$
$$u(0, y) = g(y)\ ,\ 0 \le y \le b$$
$$u(x, b) = u(a, y) = 0$$

where f (x) is a strictly increasing function and g(y) is a strictly decreasing function. Then the minimum value of u(x,y) ; $0 < x < a$, $0 < y < b$ is

A) min {f (0), g (0)} B) g (b)
C) f (0) D) g (0)

92. The first order partial differential equation satisfied by all surfaces of revolution of the form $z = F(r)$, $r = (x^2 + y^2)^{1/2}$ is

A) $x\frac{\partial z}{\partial y} = y\frac{\partial z}{\partial x}$ B) $x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}$

C) $\frac{\partial z}{\partial x}\frac{\partial z}{\partial y} = x + y$ D) $\frac{\partial z}{\partial x} + x = \frac{\partial z}{\partial y} + y$

93. A particle of mass m is constrained to move on a surface, the force of constraint is perpendicular to the surface. Then the virtual work is

A) 0
B) m
C) 2 m
D) $\frac{m}{2}$

94. The equation of motion for a simple pendulum of length l and mass of body m derived using Lagrange's equation, when θ is the angular displacement from the vertical at any time t, is given by:

A) $\ddot{\theta} - \frac{g}{l}\sin\theta = 0$
B) $\ddot{\theta} + \frac{l}{g}\sin\theta = 0$
C) $\ddot{\theta} + \frac{g}{l}\sin\theta = 0$
D) $\ddot{\theta}\ \frac{l}{g}\sin\theta = 0$

95. The characteristic curves obtained from the one-dimensional heat equation $u_{xx} - u_y = 0$ are

A) the straight lines parallel to y – axis
B) the straight lines parallel to x – axis
C) x – y = c where c is a constant
D) x + y = c where c is a constant

96. The extremal of the function $I = \int_{t_1}^{t_2} (x\dot{y} - y\dot{x})\,dt$ is a

A) Straight line
B) Parabola
C) Ellipse
D) Circle

97. The integral equation which satisfies the differential equation $y^{II}(x) = x$ and the initial conditions $y(0) = y_0$ and $y^{I}(0) = y_0^1$ is

A) $y(x) = \int_0^x (\xi - x)\,\xi\,d\xi + y_0^1 x + y_0$

B) $y(x) = \int_0^x (x - \xi)\,\xi\,d\xi + y_0^1 + y_0 x$

C) $y(x) = \int_x^0 (\xi - x)\,\xi\,d\xi + y_0^1 x + y_0$

D) $y(x) = \int_x^0 (x - \xi)\,\xi\,d\xi + y_0^1 + y_0 x$

98. The number of characteristic numbers for which the integral equation y (x) = $\lambda\int_0^1 (1-3x\xi)y(\xi)\,d\xi + F(x)$ has nontrivial solutions is

A) 0 B) 1 C) 2 D) ∞

99. The iterative solution of the integral equation $y(x) = \lambda\int_0^1 x\xi y(\xi)\,d\xi + 1$ is

A) $y(x) = 1 - x\left(\frac{\lambda}{2} + \frac{\lambda^2}{6} + \frac{\lambda^3}{18} + \ldots\right)$

B) $y(x) = 1 + x\left(\frac{\lambda}{2} - \frac{\lambda^2}{6} + \frac{\lambda^3}{18} + \ldots\right)$

C) $y(x) = x + \left(\frac{\lambda}{2} + \frac{\lambda^2}{6} + \frac{\lambda^3}{18} + \ldots\right)$

D) $y(x) = 1 + x\left(\frac{\lambda}{2} + \frac{\lambda^2}{6} + \frac{\lambda^3}{18} + \ldots\right)$

100. Which one of the following function has no Laplace transform

A) $\frac{1-e^x}{x}$

B) tan x

C) The unit step function u (x-a) defined by

$$u(x-a) = \begin{cases} 0 \text{ if } x < a \\ 1 \text{ if } x \geq a \end{cases}, \text{ where } a > 0.$$

D) The unit impulse function δ (x-a) defined by

$$\delta(x-a) = \begin{cases} \infty \text{ for } x = a \\ 0 \text{ for } x \neq a \end{cases}$$

such that $\int_0^\infty \delta(x-a)\,dx = 1 \quad (a>0)$.